

Quantum Gravity Needs Supersymmetry

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Abstract

We report on some old and new results on the quantum aspects of four-dimensional maximal supergravity, and its hypothetical ultraviolet finiteness.

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1 From Supersymmetry...

If supersymmetry is used in *string theory* (see *e.g.* [1, 2]), then it is possible to construct five seemingly consistent (M -theory descendant [3]; see *e.g.* [4] for a list of Refs.) theories in $D = 10$ space-time dimensions (see *e.g.* [5]). However, due to the large possible choice of compactification manifolds

$$\mathcal{M}_{10} \longrightarrow \mathcal{M}_4 \times X_6, \quad (1.1)$$

a multitude of $D = 4$ theories arise (it can be classified *e.g.* by a statistical approach to the vacua [6]), and it is yet a mystery how string theory could select the physical vacuum. Further models involving physical degrees of freedom living in the bulk and on branes give very interesting possibilities (see *e.g.* [7, 8, 9, 10]).

On the other hand, if supersymmetry is used in *field theory*, then one encounters the opposite situation. For *rigid* theories (*i.e.*, neglecting gravity), one can build nice but unrealistic models, explore AdS/CFT duality, and study strongly coupled gauge theories. However, when gravity is included, *at least* in the cases with $\mathcal{N} \leq 16$ supercharges, it seems that one fails to unify supersymmetry, gravity and quantum theory.

In spite of these drawbacks, as we will shortly report in the next Sections, there are some indications that for some $D = 4$ special theories of supergravity, namely $\mathcal{N} = 8$ and possibly $\mathcal{N} = 5, 6$, “miracles” occur due to unexpected cancellations in multi-loop perturbative calculations [11, 12, 13, 14, 15, 16, 17, 20, 21].

Arguments based on M -theory compactifications [22, 23, 24] and E_7 invariance [20] prevent counterterms only up to a finite loop order. On the other hand, analysis based on possible divergences in the supefield light-cone formulation may indicate the absence of counterterms at all loops [14, 17].

In October 1976, S.F. was invited by John Schwarz to visit Caltech and to give a seminar on the construction of the first supersymmetric theory of gravity in $D = 4$ (named $\mathcal{N} = 1$ *supergravity* [18, 19]); at that time, the first extended ($\mathcal{N} = 2$) supergravity had just been completed [25]. Murray Gell-Mann, during an enlightening conversation in his office, remarked that if $D = 4$ higher- \mathcal{N} supergravities existed, then $\mathcal{N} = 8$ would be the supergravity theory with maximal supersymmetry [26].

Today, after 35 years, we are still struggling with $\mathcal{N} = 8$ *maximal* supergravity, its connections to superstring and M -theory, its hypothetical perturbative finiteness and its non-perturbative completion(s).

2 ...to Maximal Supergravity...

As anticipated by Gell-Mann, $\mathcal{N} = 8$ supergravity [27, 28] is the theory with the largest possible amount of supersymmetry for particles with spin $s \leq 2$ in $D = 4$ (namely, no higher spin fields in the massless spectrum). Indeed, in supersymmetric gravity theories with \mathcal{N} -extended supersymmetry, the massless particle content is given by

$$\binom{\mathcal{N}}{k} \equiv \frac{\mathcal{N}!}{k! (\mathcal{N} - k)!} \text{ particles of helicity } \lambda = 2 - \frac{k}{2}, \quad (2.1)$$

where $k_{\max} = \mathcal{N}$, and $\mathcal{N} \leq 8$ if $|\lambda| \leq 2$ is requested.

One possible approach to maximal supergravity is to consider it as it comes from M -theory restricted to the massless sector. The problem is that this theory, even if preserving maximal $\mathcal{N} = 8$ supersymmetry (corresponding to $32 = 8 \times 4$ supersymmetries), is *not* uniquely defined, because of the multiple choice of internal compactification manifolds and corresponding duality relations:

$$\begin{aligned}
\text{I. } M_{11} &\longrightarrow M_4 \times T_7 && (GL^+(7, \mathbb{R}) \text{ and } SO(7) \text{ manifest}); \\
\text{II. } M_{11} &\longrightarrow AdS_4 \times S^7 && (SO(8) \text{ manifest, gauged}); \\
\text{III. } M_{11} &\longrightarrow M_4 \times T_{7,\mathcal{R}} && (SL(8, \mathbb{R}) \text{ and } SO(8) \text{ manifest}),
\end{aligned} \tag{2.2}$$

where T_7 is the 7-torus and S^7 is the 7-sphere. $T_{7,\mathcal{R}}$ denotes the case in which, according to Cremmer and Julia [27], the dualization of 21 vectors and 7 two-forms makes $SL(8, \mathbb{R})$ (in which $GL^+(7, \mathbb{R})$ is maximally embedded) manifest as maximal non-compact symmetry of the Lagrangian. Note that in case **III** one can further make $E_{7(7)}$ (and its maximal compact subgroup $SU(8)$) manifest *on-shell*, by exploiting a Cayley transformation supplemented by a rotation through $SO(8)$ gamma matrices on the vector 2-form field strengths [27, 29].

The fundamental massless fields (and the related number \sharp of degrees of freedom) of M -theory in $d = 11$ flat space-time dimensions are [30]

$$\begin{aligned}
g_{\mu\nu} \text{ (graviton)} : & \quad \sharp = \frac{(d-1)(d-2)}{2} - 1, && \text{in } d = 11 : \sharp = 44; \\
\Psi_{\mu\alpha} \text{ (gravitino)} : & \quad \sharp = (d-3)2^{(d-3)/2}, && \text{in } d = 11 : \sharp = 128; \\
A_{\mu\nu\rho} \text{ (three-form)} : & \quad \sharp = \frac{(d-2)(d-3)(d-4)}{3!}, && \text{in } d = 11 : \sharp = 84.
\end{aligned} \tag{2.3}$$

Because a $(p+1)$ -form (“Maxwell-like” gauge field) A_{p+1} couples to p -dimensional extended objects, and its “magnetic” dual B_{d-p-3} couples to $(d-p-4)$ -dimensional extended objects, it follows that the fundamental (massive) objects acting as sources of the theory are $M2$ - and $M5$ -branes.

In the formulation **III** of (2.2) [27], the gravitinos ψ_I and the gauginos χ_{IJK} respectively have the following group theoretical assignment¹ (I in **8** of $SU(8)$):

$$\text{theory III [27]} : \begin{cases} \psi_I : \underset{\mathbf{8}}{SO(7)} \subset \underset{\mathbf{8}_s}{SO(8)} \subset \underset{\mathbf{8}}{SU(8)}; \\ \chi_{IJK} : \underset{\mathbf{8+48}}{SO(7)} \subset \underset{\mathbf{56}_s}{SO(8)} \subset \underset{\mathbf{56}}{SU(8)}. \end{cases} \tag{2.4}$$

On the other hand, the 70 scalar fields arrange as

$$\text{theory III [27]} : s = 0 \text{ dofs} : \underset{(\sharp=70)}{SO(7)} \subset \underset{\mathbf{1+7+21+35}}{SO(8)} \subset \underset{\mathbf{35}_v+\mathbf{35}_c}{SU(8)}, \tag{2.5}$$

¹As evident from (2.4), we use a different convention with respect to [31] (see *e.g.* Table 36 therein). Indeed, we denote as $\mathbf{8}_v$ of $SO(8)$ the irrep. which decomposes into $\mathbf{7} + \mathbf{1}$ of $SO(7)$, whereas the two spinorial irreps. $\mathbf{8}_s$ and $\mathbf{8}_c$ both decompose into $\mathbf{8}$ of $SO(7)$. The same change of notation holds for $\mathbf{35}$ and $\mathbf{56}$ irreps..

where **70** is the rank-4 completely antisymmetric irrep. of $SU(8)$, the maximal compact subgroup of the U -duality group $E_{7(7)}$ (also called \mathcal{R} -symmetry). It follows that scalars parameterize a non-compact coset manifold $\frac{G}{SU(8)}$. Indeed, the $SU(8)$ under which both the scalar fields and the fermion fields transform is the “local” $SU(8)$, namely the stabilizer of the scalar manifold. On the other hand, also a “global” $SU(8)$ (\mathcal{R} -symmetry group) exists, under which the vector 2-form self-dual/anti-self-dual field strengths transform. Roughly speaking, the physically relevant group $SU(8)$ is the diagonal one in the product $SU_{\text{local}}(8) \times SU_{\text{global}}(8)$ (see also discussion below).

Remarkably, there exists an *unique* simple, non-compact Lie group with real dimension $70 + 63 = 133$ and which embeds $SU(8)$ as its *maximal* compact subgroup: this is the real, non-compact split form $E_{7(7)}$ of the exceptional Lie group E_7 , thus giving rise to the symmetric, rank-7 coset space

$$\frac{E_{7(7)}}{SU(8)/\mathbb{Z}_2}, \quad (2.6)$$

which is the scalar manifold of $\mathcal{N} = 8$, $D = 4$ supergravity (\mathbb{Z}_2 is the kernel of the $SU(8)$ -representations of even rank; in general, spinors transform according to the double cover of the stabilizer of the scalar manifold; see *e.g.* [32, 33]).

$E_{7(7)}$ acts as electric-magnetic duality symmetry group [34], and its maximal compact subgroup $SU(8)$ has a chiral action on fermionic as well as on (the vector part of the) bosonic fields. While the chiral action of $SU(8)$ on fermions directly follows from the chirality (complex nature) of the relevant irreps. of $SU(8)$ (as given by Eq. (2.4)), the chiral action on vectors is a crucial consequence of the electric-magnetic duality in $D = 4$ space-time dimensions. Indeed, this latter allows for “self-dual / anti-self-dual” complex combinations of the field strengths, which can then fit into complex irreps. of the stabilizer H of the coset scalar manifold G/H itself. For the case of maximal $\mathcal{N} = 8$ supergravity, the relevant chiral complex irrep. of $H = SU(8)$ is the rank-2 antisymmetric **28**.

Note that if one restricts to the $SL(8, \mathbb{R})$ -covariant sector, the chirality of the action of electric-magnetic duality is spoiled, because the maximal compact subgroup of $SL(8, \mathbb{R})$, namely $SO(8)$, has not chiral irreps.

Composite (sigma model G/H) anomalies can arise in theories in which G has a maximal compact subgroup with a *chiral* action on bosons and/or fermions (see *e.g.* [35, 36, 13]). Surprising cancellations among the various contributions to the composite anomaly can occur as well. An example is provided by $\mathcal{N} = 8$, $d = 4$ supergravity itself, in which standard anomaly formulæ yield the remarkable result [36, 13]

$$3Tr_{\mathbf{8}}X^3 - 2Tr_{\mathbf{28}}X^3 + Tr_{\mathbf{56}}X^3 = (3 - 8 + 5)Tr_{\mathbf{8}}X^3 = 0, \quad (2.7)$$

where X is any generator of the Lie algebra $\mathfrak{su}(8)$ of the *rigid* (*i.e.* *global*) $SU(8)$ group (\mathcal{R} -symmetry). In light of the previous considerations, the first and third contributions to (2.7) are due to fermions: the 8 gravitinos ψ_A and the 56 spin- $\frac{1}{2}$ fermions χ_{ABC} , respectively, whereas the second contribution is due to the 28 chiral vectors. Note that, for the very same reason, the *local* $SU(8)$ (stabilizer of the non linear sigma-model of scalar fields), under which only fermions do transform², would be *anomalous* [35]. In an analogous way, in [36] it was discovered that $\mathcal{N} = 6$ and $\mathcal{N} = 5$ “pure” supergravities are *composite anomaly-free*, whereas $\mathcal{N} \leq 4$ theories are not.

²Also scalar fields transform under *local* $SU(8)$, but they do not contribute to the composite anomaly, because they sit in the *self-real* (and thus *non-chiral*) rank-4 antisymmetric irrep. **70** of $SU(8)$.

3 ...and its UV Perturbative Finiteness?

At homotopical level, the following holds:

$$E_{7(7)} \cong (SU(8)/\mathbb{Z}_2) \times \mathbb{R}^{70}, \quad (3.1)$$

implying that the two group manifolds have the same De Rham cohomology. This is a key result, recently used in [13] to show that the aforementioned absence of $SU(8)$ current anomalies yield to the absence of anomalies for the non-linearly realized $E_{7(7)}$ symmetry, thus implying that the $E_{7(7)}$ continuous symmetry of classical $\mathcal{N} = 8$, $d = 4$ supergravity is preserved at all orders in perturbation theory (see *e.g.* [11, 14, 37, 15, 16, 17, 20]). This implies the perturbative finiteness of supergravity at least up to seven loops; Bern, Dixon *et al.* explicitly checked the finiteness up to four loops included [11] (computations at five loops, which might be conclusive, are currently in progress; for a recent review, see *e.g.* [21]).

A puzzling aspect of these arguments is that string theory certainly violates continuous $E_{7(7)}$ symmetry at the perturbative level, as it can be easily realized by considering the dilaton dependence of loop amplitudes (see *e.g.* [20]). However, this is not the case for $\mathcal{N} = 8$ supergravity. From this perspective, two (perturbatively finite) theories of quantum gravity would exist, with 32 local supersymmetries; expectedly, they would differ at least in their non-perturbative sectors, probed *e.g.* by black hole solutions. String theorists [38, 39, 40] claim that $\mathcal{N} = 8$, $d = 4$ supergravity theory is probably not consistent at the non-perturbative level. From a purely $d = 4$ point of view, their arguments could be overcome by excluding from the spectrum, as suggested in [37], black hole states which turn out to be singular or ill defined if interpreted as purely four-dimensional gravitational objects. Inclusion of such singular states (such as $\frac{1}{4}$ -BPS and $\frac{1}{2}$ -BPS black holes) would then open up extra dimensions, with the meaning that a non-perturbative completion of $\mathcal{N} = 8$ supergravity would lead to string theory [38]. Extremal black holes with a consistent $d = 4$ interpretation may be defined as having a Bertotti-Robinson [41] $AdS_2 \times S^2$ near-horizon geometry, with a non-vanishing area of the event horizon. In $\mathcal{N} = 8$ supergravity, these black holes are³ $\frac{1}{8}$ -BPS or non-BPS (for a recent review and a list of Refs., see *e.g.* [44]). The existence of such states would in any case break the $E_{7(7)}(\mathbb{R})$ continuous symmetry, because of Dirac-Schwinger-Zwanziger dyonic charge quantization conditions.

The breaking of $E_{7(7)}(\mathbb{R})$ into an arithmetic subgroup $E_{7(7)}(\mathbb{Z})$ [45] would then manifest only in exponentially suppressed contributions to perturbative amplitudes (see *e.g.* the discussion in [13], and Refs. therein), in a similar way to instanton effects in non-Abelian gauge theories. Indeed, as for the $SL(2, \mathbb{R})$ symmetry of Type IIB superstrings in $D = 10$ [22], the continuous symmetry is believed to be broken down to discrete $E_{7(7)}(\mathbb{Z})$ by non-perturbative effects like instantons [46]. The breaking of $E_{7(7)}(\mathbb{R})$ to $E_{7(7)}(\mathbb{Z})$ is instrumental to setting a uniform mass-gap, given by Planck Mass $M_{Pl} = \sqrt{\hbar c/G_N} = 1.22 \times 10^{19} GeV/c^2$, everywhere inside the moduli space $E_{7(7)}/SU(8)$, for all regular BPS states with $\mathcal{I}_4 \neq 0$ where [47, 48]

$$\mathcal{I}_4 = \mathbb{K}_{MNPQ} Q^M Q^N Q^P Q^Q \quad (3.2)$$

³We also remark that these are the only black holes for which the *Freudenthal duality* [42, 43] is well defined.

is the *quartic* Cartan invariant of $E_{7(7)}$ [27] and Q^M is a 56-dimensional vector of ‘bare’ quantized electric and magnetic charges with respect to the 28 $U(1)$ gauge groups.

In [37], after taking into account charge quantization and assuming that the perturbative theory be UV finite to all orders, the plausibility of a *non-perturbative* completion of genuinely $D = 4$, $\mathcal{N} = 8$ supergravity, only including regular black hole states with $\mathcal{I}_4 \neq 0$ and excluding all singular states with $\mathcal{I}_4 = 0$, was proposed. This proposal has some analogy with $\mathcal{N} = 4$ super Yang-Mills (SYM) theory in $D = 4$, decoupled from gravity and other stringy interactions. Even after including non-perturbative effects, $\mathcal{N} = 4 = 4$ SYM in $D = 4$ should not be thought of as a compactification of Type I or Heterotic strings, that contain the same massless states but differ by the massive completion, but rather in terms of the AdS/CFT correspondence [49]. Analogously, pure $D = 4$ $\mathcal{N} = 8$ supergravity, including *regular* non-perturbative states, may be disconnected from toroidal compactifications of Type II superstrings, that unavoidably give rise to 1/2 BPS states with $\mathcal{I}_4 = 0$. The fact that all known $\mathcal{N} = 8$ supergravity perturbative amplitudes could be expressed in terms of $\mathcal{N} = 4$ SYM amplitudes in the superconformal phase, where the latter enjoys 32 supersymmetries (16 of Poincaré type plus 16 superconformal), might be more than an analogy in this respect.

The conjectured UV finiteness of $\mathcal{N} = 8$ supergravity, associated with continuous $E_{7(7)}$ symmetry, has been questioned by Green, Ooguri, Schwarz in [50], where non-decoupling of BPS states from four-dimensional $\mathcal{N} = 8$ supergravity was discussed. The main conclusion of [50] was that the $\mathcal{N} = 8$ supergravity limit of string theory does not exist in four dimensions, irrespective of whether or not the perturbative approximation is free of UV divergences. String theory adds to the 256 massless states of four-dimensional $\mathcal{N} = 8$ supergravity an infinite tower of states, such as Kaluza-Klein momenta and monopoles, wound strings and wrapped branes.

As mentioned, classical solutions of the $\mathcal{N} = 8$ version of non-linear Einstein equations including stable, zero temperature, extremal, BPS and non-BPS charged black holes [51], should play a crucial role in defining the quantum features of the theory. For appropriate choices of the charges, these black holes can be viewed as smooth solitons interpolating between flat Minkowski space-time at infinity and Bertotti-Robinson $AdS_2 \times S^2$ geometry [41] near the horizon. The asymptotic values of the scalar fields are largely arbitrary and determine the ADM mass M [52] for given charges Q^M . Thanks to the *attractor mechanism* [53], their near-horizon values are determined in terms of the charges. The entropy of a black hole in $\mathcal{N} = 8$ supergravity is related to the horizon area by the Bekenstein-Hawking formula

$$S_{BH} = \frac{1}{4} A_H = \pi \sqrt{|\mathcal{I}_4|}, \quad (3.3)$$

and $\mathcal{N} = 8$ attractors were studied in [54].

Within this framework, the ADM mass of an $\mathcal{N} = 8$ extremal black hole depends on its charges and on the asymptotic values of the scalar fields, both transforming under $E_{7(7)}$ symmetry:

$$M_{BH} = M_{ADM}(Q, \phi). \quad (3.4)$$

A manifestly $E_{7(7)}$ covariant expression for the mass is related to the maximal eigenvalue of the central charge matrix [45],[55]

$$M_{ADM}^2(Q, \phi) \geq \text{Max}_i \{|Z_i(Q, \phi)|^2\}, \quad (3.5)$$

where $Z_i(Q, \phi)$, $i = 1, 2, 3, 4$, are the four (skew) eigenvalues [56] of the ‘dressed’ central charge matrix $Z_{AB}(Q, \phi)$. Indeed, the positive hermitian matrix $\mathcal{H}_A{}^B = Z_{AC}\bar{Z}^{BC}$ has four *real positive eigenvalues* $|Z_i(Q, \phi)|^2 = \lambda_i$ which, without any loss of generality, can be put in decreasing order $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$. The BPS condition requires that the ADM mass be exactly equal to the largest eigenvalue of the central charge matrix, i.e. it saturates the bound (3.5). On the other hand, non-BPS extremal black holes have a mass which is strictly larger than the largest eigenvalue of the central charge matrix; in such a case $\mathcal{I}_4 < 0$, and the non-BPS extremal black hole geometry is regular and its mass is never zero : $S_{BH, non-BPS} = \pi\sqrt{-\mathcal{I}_4}$. Thus, it follows that for regular black holes ($\mathcal{I}_4 \neq 0$) the ADM mass is bounded from below as a function in the moduli space; no “massless black holes” can exist in this case, contrarily to the $\mathcal{N} = 2$ [57, 40] (and $\mathcal{N} = 4$ [37]) cases.

In the $SU(8)$ covariant ‘dressed’ central charge basis, the quartic invariant (3.2) in the area/entropy formula (3.3) reads

$$\mathcal{I}_4(Q, \phi) = Tr[(Z\bar{Z})^2] - \frac{1}{4}[Tr(Z\bar{Z})]^2 + 8\text{Re}Pf(Z), \quad (3.6)$$

where $Pf(Z)$ denotes the *Pfaffian* of Z_{AB} ; it should be remarked that each $SU(8)$ invariant term in this expression depends on the moduli, but nevertheless the total expression is moduli independent due to $E_{7(7)}$ symmetry.

As a final comment, it should be stressed that if the perturbative theory has some - yet unknown - UV divergences [23, 24], the analysis of the massless black hole solutions and the proposal put forward in [37] may require modifications. However, if there are no UV divergences in perturbation theory, corrections to the analysis of the space-time properties of these states are not expected. In particular, the states in [50] have been shown to be singular in [37]. There are two reasonable options: *i*) these states may be consistently excluded from the four-dimensional theory and therefore do not affect UV properties of $\mathcal{N} = 8$, $D = 4$ supergravity; *ii*) they can be proven to be required in $D = 4$ and affect the perturbative theory.

While some evidence for the plausibility of the first option was given in [37], a conclusive answer has not been given yet.

4 Conclusive Remarks

If $\mathcal{N} = 8$, $D = 4$ supergravity turns out to be UV perturbatively finite, according to Bern *et al.* [11, 21], it is not only due to maximal supersymmetry and to perturbatively unbroken $E_{7(7)}$ symmetry, but also to other reasons.

One of these is the “*double copy*” structure [58], which implies a relation, not only kinematical but also dynamical, between the square of the $\mathcal{N} = 4$ super Yang-Mills amplitudes and $\mathcal{N} = 8$ amplitudes. At loop level, the “double copy” properties of amplitudes have been extended to supergravity theories with $\mathcal{N} \geq 4$; in this case, one copy is given by $\mathcal{N} = 4$ Yang-Mills and the other copy is an $\mathcal{N} = 0, 1, 2$ Yang-Mills gauge theory, thus giving rise to $\mathcal{N} = 4, 5, 6$ supergravity theories. From the analysis of divergences, one is led to conclude that $\mathcal{N} = 6$ and $\mathcal{N} = 5$ supergravity may be UV finite (if $\mathcal{N} = 8$ is), while $\mathcal{N} = 4$ probably is not. It is worth remarking that these results are in agreement with

the “composite anomaly” arguments [36, 13] for which $\mathcal{N} = 5, 6$ do not exhibit duality anomalies, while $\mathcal{N} = 4$ does.

Another interesting aspect [21] which should be implied by UV finiteness of $N = 8, 6, 5$ supergravity in $D = 4$ dimensions is that their *gauged* versions should be possibly UV finite, as well. Roughly speaking, this is related to the fact that *gauging* may be regarded as a spontaneous soft breaking of an unbroken gauge symmetry, and UV properties should not be affected by such a spontaneous breaking, as it happens in the Standard Model of electro-weak interactions.

We have already commented on the difficulties and subtleties related to the question of whether a point-like non-perturbative completion of $\mathcal{N} = 8$ supergravity exists. Single-centered BPS black hole states preserving a large fraction of supersymmetry (1/2 or 1/4) are singular in Einstein theory of gravity, and thus physically unacceptable. However, such states may be “*confined*”, *i.e.* they may only exist as building blocks of multi-centered black hole configurations; the viability and physical meaning of such a phenomenon are currently unexplored issues. On the other hand, from a superstring theory perspective such singular states are just an indication that the fundamental theory is ill-defined, and that extra dimensions *and/or* a “*non-local*” structure with minimal length open up in the non-perturbative quantum regime.

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